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Spatiotemporally localized solitons in resonantly absorbing Bragg reflectors

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We predict the existence of multidimensional solitons that are localized in both space and time (''light bullets'') in two- and three-dimensional self-induced-transparency media embedded in a Bragg grating. These fully stable light bullets suggest new possibilities of signal transmission control and self-trapping of light.

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''Light bullets'' are multidimensional solitons that are localized in both space and time. In the last decade they have been theoretically investigated in various nonlinear optical media [1–5], and recently, the first experimental observation of a quasi-two-dimensional (2D) bullet was reported [6]. A promising candidate for observation of fully 2D and 3D light bullets is a self-induced-transparency (SIT) medium [7]. SIT involves the undistorted and unattenuated propagation of an electromagnetic pulse in a medium consisting of near-resonant two-level atoms, irrespective of the carrier-frequency detuning from resonance [8,9]. In Ref. [7] we had predicted that uniform 2D and 3D SIT-media can support stable light bullets. Here we extend this investigation to SIT media which are embedded in a layered dielectric structure. A well-known example of such a structure is a resonantly absorbing Bragg reflector (RABR), a 1D periodic grating composed of thin layers of SIT media separated by half-wavelength nonabsorbing dielectric layers. Pulse transmission through a RABR can produce SIT in the band gap of the structure [10,11] and gives rise to various types of 1D soliton dynamics [12,13]. This prompted us to search for multidimensional solitons in the form of light bullets in higher-dimensional Bragg gratings. Our main finding is that for any Bragg reflectivity a 2D RABR can support stable light bullets, which are closely related to those in uniform SIT media [7].

A Bragg reflector consists of a linear optical medium with a periodic modulation of the index of refraction along the z-direction, given by [14]

$$n^2(z)=n_0^2[1+a_1\cos(2k_c z)]. \tag{1}$$

Here n_0 and a_1 are constants and $k_c=\omega_c/c$, with ω_c the central frequency of the band gap. A resonantly absorbing Bragg reflector (RABR) is then constructed by placing very thin layers (much thinner than $1/k_c$) of two-level atoms, whose resonance frequency is close to ω_c, at the maxima of this modulated index of refraction. We study the propagation of an electromagnetic wave with frequency close to ω_c through a 2D RABR. Due to Bragg reflections, the electric field is decomposed into forward and backward propagating components which satisfy the 2D equations [15]

$$-i\Sigma^+_{\tau xx}+i\Sigma^-_{zxx}+\Sigma^+_{\tau\tau}-\Sigma^+_{zz}+\eta\Sigma^+_{xx}+\eta^2\Sigma^+-2\mathcal{P}_\tau-2i\eta\mathcal{P}=0, \tag{2a}$$

$$-i\Sigma^-_{\tau xx}+i\Sigma^+_{zxx}+\Sigma^-_{\tau\tau}-\Sigma^-_{zz}-\eta\Sigma^-_{xx}+\eta^2\Sigma^-+2\mathcal{P}_z=0, \tag{2b}$$

$$\mathcal{P}_\tau+i\Delta\Omega\mathcal{P}-\Sigma^+W=0, \tag{2c}$$

$$W_\tau+\frac{1}{2}(\Sigma^{+*}\mathcal{P}+\Sigma^+\mathcal{P}^*)=0. \tag{2d}$$

Here $\Sigma^\pm\equiv 2\tau_0\mu n_0(\mathcal{E}_F\pm\mathcal{E}_B)/\hbar$, where $\mathcal{E}_F$ and $\mathcal{E}_B$ denote the slowly varying amplitudes of respectively the forward and backward propagating field, and $\tau_0\equiv n_0\mu^{-1}\sqrt{\hbar/2\pi\omega_c\rho_0}$, with μ the transition dipole moment and ρ_0 the density of the two-level atoms. $\mathcal{P}$ and W denote the slowly varying amplitudes of the polarization and inversion, respectively, z and x are the longitudinal and transverse coordinates (in units of the effective absorption length α_{eff}), τ denotes the time (in units of the input pulse duration τ_p) and $\Delta\Omega$ the detuning of the carrier frequency ω_0 from the central atomic-resonance frequency. The Fresnel number F ($F>0$), which governs the transverse diffraction in 2D and 3D propagation, has been incorporated in x [16]. We have neglected polarization dephasing and inversion decay, considering pulse durations that are short on the time scale of these relaxation processes. Equations (2) are then compatible with the local constraint $|\mathcal{P}|^2+W^2=1$, which corresponds to conservation of the Bloch vector [17]. The parameter η is the central quantity characterizing a RABR. It is defined as the ratio of the two-level atom absorption length to the Bragg reflection length and can be expressed as $\eta=a_1\omega_c\tau_0/4$. In 1D, the solution of Eq. (2) is given by

$$\Sigma^+=2\,\alpha\,\mathrm{sech}\,\Theta(\tau,z)e^{i\eta M\tau+i\eta Nz+i\phi}, \tag{3a}$$

$$\Sigma^-=-2\,\sqrt{\alpha^2+2}\,\mathrm{sech}\,\Theta(\tau,z)e^{i\eta M\tau+i\eta Nz+i\phi}, \tag{3b}$$

$$\mathcal{P}=2\,\mathrm{sech}\,\Theta(\tau,z)\tanh\Theta(\tau,z)e^{i\eta M\tau+i\eta Nz+i\phi}, \tag{3c}$$

$$W=\mathrm{sech}^2\,\Theta(\tau,z)-\tanh^2\Theta(\tau,z), \tag{3d}$$

with $\Theta(\tau,z)\equiv\alpha\tau+\sqrt{\alpha^2+2}\,z+\Theta_0$, $M\equiv-(\alpha^2+1)$, $N\equiv-\alpha\sqrt{\alpha^2+2}$ and for $\Delta\Omega=\eta(\alpha^2+1)$ [18]. Here α and Θ_0 denote arbitrary real parameters. The shape of the fields Σ^+ and Σ^- in equations (3) is very similar to the sine-Gordon (SG) soliton in a uniform 1D SIT medium, which is given by

$$\mathcal{E}(\tau,z)=2\alpha\,\mathrm{sech}(\alpha\tau-z/\alpha+\Theta_0). \tag{4}$$

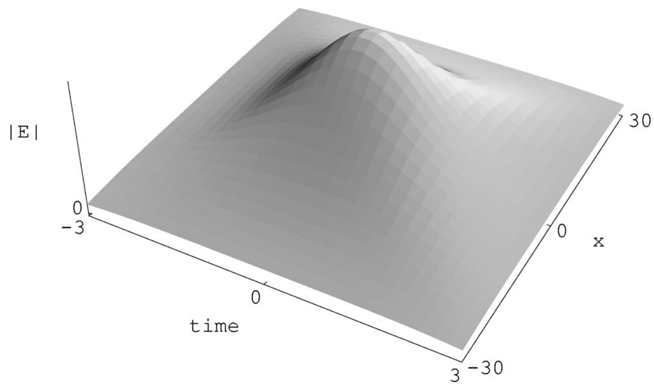

FIG. 1. Forward-propagating electric field in the 2D ''light bullet'' in a RABR, $|\mathcal{E}_F|$, as a function of time τ (in units of the input pulse duration τ_p) and transverse coordinate x (in units of the effective absorption length $\alpha_{\rm eff}$) after propagating the distance $z=1000$. Parameters used correspond to $\alpha=1$, $C=0.1$ and $\Theta_0=-1000$. The field is scaled by the constant $\hbar/(4\tau_0\mu n_0)$.

Inspired by this analogy and the fact that there exists a light bullet in a uniform 2D SIT medium which reduces to the SG soliton (4) in one dimension [7], we now search for a light-bullet solution of the 2D RABR equations (2), which reduces to the soliton (3) in 1D. This solution exists and is given by

$$\Sigma^+=2\,\alpha\,\sqrt{\operatorname{sech}\Theta_1\operatorname{sech}\Theta_2}\,e^{i\eta M\tau+i\eta Nz+i\pi/4}, \tag{5a}$$

$$\Sigma^-=-2\,\sqrt{\alpha^2+2}\,\sqrt{\operatorname{sech}\Theta_1\operatorname{sech}\Theta_2}\,e^{i\eta M\tau+i\eta Nz+i\pi/4}, \tag{5b}$$

$$\mathcal{P}=\sqrt{\operatorname{sech}\Theta_1\operatorname{sech}\Theta_2}\bigg\{(\tanh\Theta_1+\tanh\Theta_2)^2 +\frac{1}{4}\alpha^2C^4[(\tanh\Theta_1-\tanh\Theta_2)^2 -2(\operatorname{sech}^2\Theta_1+\operatorname{sech}^2\Theta_2)]^2\bigg\}^{1/2}e^{i\eta M\tau+i\eta Nz+i\nu}, \tag{5c}$$

$$W=[1-|\mathcal{P}|^2]^{1/2}, \tag{5d}$$

with $\Theta_1(\tau,z)\equiv\alpha\tau+\sqrt{\alpha^2+2}\,z+\Theta_0+Cx$, $\Theta_2(\tau,z)\equiv\alpha\tau+\sqrt{\alpha^2+2}\,z+\Theta_0-Cx$, the phase $\nu\equiv\arctan[\mathrm{Im}(\mathcal{P})/\mathrm{Re}(\mathcal{P})]$ and C a real constant. Equations (5) form our central result. They satisfy Eqs. (2a) and (2b) exactly and Eqs. (2c) and (2d) to order $|\alpha|C^2$, which requires that $|\alpha|C^2\ll1$. They are valid for *arbitrary* η, so both for weak ($\eta\ll1$) and strong ($\eta>1$) reflectivities of the Bragg grating, provided the detuning $\Delta\Omega$ remains small with respect to the gap frequency ω_c, $\Delta\Omega\ll\omega_c\Leftrightarrow\eta\ll\omega_c/(\alpha^2+1)$. Comparison with numerical simulations of Eqs. (2), using (5) as an initial ansatz, test this approximation and show that it is indeed a good approximation to the exact solution, which predicts the shape of the bullet still within 98% accuracy after propagating a large distance, typically $z\sim1000$, as in Fig. 1.

Equations (5) form the extension of the light-bullet solution in a uniform SIT medium, Eq. (8) in Ref. [7], to a RABR and reduce to the latter for $\eta=0$ [19]. Comparing the two, the presence of the Bragg grating adds a second field to the uniform solution, due to the retro-reflections, and leads to additional phasefactors in the fields and polarization.

Also the 3D axisymmetric light bullet that was found in a uniform SIT medium [7] has a counterpart in a 3D RABR. The 3D RABR is described by equations (2) with $\Sigma_{xx}^{+(-)}$ replaced by $\Sigma_{rr}^{+(-)}+r^{-1}\Sigma_r^{+(-)}$, where $r\equiv\sqrt{x^2+y^2}$ is the transverse radial coordinate. In the limit of large r, for $r\gg1/|C|$, an approximate 3D light bullet solution exists which is of the same form as Eq. (5), with x replaced by r and is valid for $|\alpha|C^2\ll1$. It is in agreement with results of simulation of the 3D equations using this solution as an initial ansatz (deviations $\sim5\%$), as is also the case for a uniform medium.

We have checked that the light bullet (5) is stable for all values of z by means of the Vakhitov-Kolokolov stability criterion, which says that a necessary condition for stability is a positive derivative of the norm of the total light bullet field with respect to the propagation constant [20]. This stability is also seen numerically (checked up to $z\sim10^4$).

It is straightforward to see that 2D light bullets of the variable-separated form $\Sigma^{+(-)}\sim f(\tau,z)g(x)$, as we found in a SIT medium with transversely-varying index of refraction [21], do not exist in a 2D RABR. Substituting this form into Eqs. (2a) and (2b) only yields a solution of the form $\Sigma^{+(-)}\sim e^{iA\tau}e^{iBx}$, with A and B constants, which does not correspond to a light bullet.

Experimentally, present-day nanolithography techniques allow for fabrication of dielectric structures with layer thicknesses on the order of a few atomic layers [22,23], and the study of light-matter interactions in such structures has developed into a vast research area [24]. Realization of light bullets in the resonantly absorbing gratings discussed above presents a new experimental challenge in this field and would be the first demonstration of localized multidimensional solitons in optical Bragg grating structures. In order to realize a RABR, quantum wells embedded in a semiconductor structure with a spatially varying index of refraction may be used. The excitons in the quantum wells then act as effective two-level systems, and typical system parameters are: the average refractive index $n_0\sim3.6$, the central gap frequency $\omega_c\sim10^{15}\,\mathrm{s}^{-1}$, the density of the excitons $\rho_0\sim10^{15}-10^{16}\,\mathrm{cm}^{-3}$, and the characteristic absorption time and length $\tau_0\sim10^{-13}-10^{-12}$ s and $\alpha_{\rm eff}\sim10^4-10^5\,\mathrm{m}^{-1}$ [13,23]. The parameter η can vary from 0 to 100 and the detuning $\Delta\Omega\sim10^{12}-10^{13}\,\mathrm{s}^{-1}$, which satisfies the condition $\Delta\Omega\ll\omega_c$. Using an incident optical pulse generated by a laser with pulse duration $\tau_p<0.1$ ns, one has $\omega_0>10^{10}$ s^{-1}. The incident pulse should be of uniform transverse intensity and satisfy $\alpha_{\rm eff}d^2/\lambda_0<1$, where λ_0 and d are its carrier wavelength and diameter respectively, in order to include transverse diffraction [25]. For $\lambda_0\sim10^{-4}$ m one thus requires $d<10^{-4}$ m, or a transverse medium size $L_x\sim1-10\,\mu$m. The parameter α in the light-bullet solution (5), which determines the amplitude of the bullet and its decay in time and z, corresponds to $\alpha=\sqrt{v_z\alpha_{\rm eff}\tau_p/(1-v_z\alpha_{\rm eff}\tau_p)}$, with v_z the longitudinal velocity of the pulse in the medium, and can thus be controlled by the incident pulse duration and velocity. For atomic-gas media one typically has $\alpha\sim0.1-10$. The parameter β is similarly controlled and given by

$\beta \sim \kappa_x L_x \sim 1-10$, with κ_x the wave vector component along the transverse direction x. The light bullets as depicted in Fig. 1 decay on a transverse length scale of ~ 1 μm and time scale $\sim 10^{-13}$ s. The dephasing time discussed in Ref. [23] is also $\sim 10^{-13}$ s, but cryogenic conditions can extend this well into the nsec range [26]. The construction of suitable structures constitutes an experimental challenge.

In summary, we have studied and predicted the existence of fully stable light bullets in multidimensional resonantly absorbing Bragg reflectors. These offer the possibility to realize a novel type of filters, which stably transmit selected signal frequencies through their spectral gap and simultaneously block others. They can also be used to both spatially and temporarily localize light in certain frequency bands.

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[1] Y. Silberberg, Opt. Lett. **15**, 1281 (1990).

[2] K. Hayata and M. Koshiba, Phys. Rev. Lett. **71**, 3275 (1993); B.A. Malomed, P.D. Drummond, H. He, A. Berntson, D. Anderson, and M. Lisak, Phys. Rev. E **56**, 4725 (1997); D. Mihalache, D. Mazilu, B.A. Malomed, and L. Torner, Opt. Commun. **152**, 365 (1998); H. He and P. Drummond, Phys. Rev. E **58**, 5025 (1998).

[3] K.V. Kheruntsyan and P.D. Drummond, Phys. Rev. A **58**, 2488 (1998); **58**, R2676 (1998).

[4] D.J. Frantzeskakis, K. Hizanidis, B.A. Malomed, and C. Polymilis, Phys. Lett. A **248**, 203 (1998).

[5] G. Gottwald, R. Grinshaw, and B.A. Malomed, Phys. Lett. A **248**, 208 (1998).

[6] X. Liu, L.J. Qian, and F.W. Wise, Phys. Rev. Lett. **82**, 4631 (1999).

[7] M. Blaauboer, B.A. Malomed, and G. Kurizki, Phys. Rev. Lett. **84**, 1906 (2000).

[8] S.L. McCall and E.L. Hahn, Phys. Rev. Lett. **18**, 408 (1967); G.L. Lamb, Rev. Mod. Phys. **43**, 99 (1971).

[9] See, for a review on SIT-theory, e.g., A.A. Maimistov, A.M. Basharov, S.O. Elyutin, and Yu. M. Sklyarov, Phys. Rep. **191**, 1 (1990).

[10] B.I. Mantsyzov, Phys. Rev. A **51**, 4939 (1995).

[11] A. Kozhekin and G. Kurizki, Phys. Rev. Lett. **74**, 5020 (1995).

[12] A. Kozhekin, G. Kurizki, and B.A. Malomed, Phys. Rev. Lett. **81**, 3647 (1998).

[13] T. Opatrny, B.A. Malomed, and G. Kurizki, Phys. Rev. E **60**, 6137 (1999).

[14] In fact, $n^2(z)$ is an infinite Fourier series. For the propagation of waves whose wave vector is close to k_c, however, only the coefficient a_1 is essential; see, e.g., Ref. [13].

[15] These equations are the extension to 2D of the equations describing a 1D RABR, as given in Refs. [11–13].

[16] Bringing the Fresnel number F and wave vector $k_0 \equiv \omega_0/c$ back explicitly into Eqs. (2) requires the transformations $\mathcal{E}(\tau,z,x) \rightarrow (2/k_0)\, \mathcal{E}(\tau,z,x)$, $\mathcal{P}(\tau,z,x) \rightarrow (4/k_0^2)\, \mathcal{P}(\tau,z,x)$, $W(\tau,z,x) \rightarrow (4/k_0^2)\, W$, $\tau \rightarrow (2/k_0)\, \tau$, $z \rightarrow (2/k_0)\, z$ and $x \rightarrow \sqrt{2/(k_0 F)}\, x$.

[17] A.C. Newell and J.V. Moloney, *Nonlinear Optics* (Addison-Wesley, Redwood City, CA, 1992).

[18] A solution for $\Delta\Omega = 0$ has been derived in Ref. [11].

[19] For $\eta = 0$, Eqs. (5a), (5c), and (5d) are equivalent to Eqs. (8) in Ref. [7]. The field Σ^- becomes an auxiliary field and does not play a role in the uniform SIT medium.

[20] M.G. Vakhitov and A.A. Kolokolov, Radiophys. Quantum Electron. **16**, 783 (1973); see also the paper by B.A. Malomed *et al.* in Ref. [2].

[21] M. Blaauboer, G. Kurizki, and B.A. Malomed (unpublished).

[22] A. Yariv and P. Yeh, *Optical Waves in Crystals* (Wiley, New York, 1984).

[23] G. Khitrova, H.M. Gibbs, F. Jahnke, M. Kira, and S.W. Koch, Rev. Mod. Phys. **71**, 1591 (1999).

[24] See the *Photonic Band-Gap Bibliography*, compiled by J.P. Dowling and H.O. Everitt at www: http://home.earthlink.net/jpdowling/pbgbib.html

[25] R.E. Slusher, in *Progress in Optics*, edited by E. Wolf (North-Holland, Amsterdam, 1974), Vol. 12.

[26] C. Greiner *et al.*, Phys. Rev. A **60**, R2657 (1999).